Broken Chiral Symmetry in Hot QCD: The Pion Halo *

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The massless pion propagates in hot medium with an exponentially damped halo.

1. Introduction

It has been 34 years since Sudarshan and Marshak first derived the V–A theory [1] based on the invariance of the weak interaction Lagrangian under the chiral transformation

$$\psi(x) \rightarrow e^{i\alpha\gamma_5}\psi(x). \quad (1)$$

The pioneering work has laid the foundation for weak interaction physics and led to the development of unified theories of later years. Chiral invariance protects the fermions form acquiring mass. Observed fermion masses thus must be attributed to spontaneous breaking of chiral symmetry [2].

In this talk, I report on the state of chiral symmetry in QCD. At $T = 0$, the QCD vacuum is known to be a chiral broken ground state. This is seen in the non-vanishing of the characteristic signature $\langle\bar{\psi}\psi\rangle$. Field theory calculation shows the result [3]

$$\langle\bar{\psi}\psi\rangle = -0.0398\, N_c \Lambda_c^3. \quad (2)$$

For nonzero temperatures, lattice as well as analytic calculations show that $\langle\bar{\psi}\psi\rangle$ vanishes for $T > T_c$. It is tempting to conclude from this that chiral symmetry is restored above T_c.

Our calculations show, however, that chiral symmetry remains broken at all temperatures [4]. In particular, the π in pure QCD remains massless at all temperatures [5]. And yet lattice calculations show that the π has a *non-vanishing screening* mass. Is there a conflict between the two results?

The answer is no. In a thermal environ, the pion signal travels along the light cone, with however an exponentially damped halo along the light cone [see (21) below]. The skin depth associated with the halo is controlled by the screening mass.

The controversy rather is in the conclusions drawn from the results.

2. NJL

Why is it so 'counter-intuitive' to argue that chiral symmetry is *not* restored at high temperatures?

Deeply ingrained in our intuition is the analogy with ferromagnetism. At zero temperature, the spins are all maximally lined up. As the material is heated up, these spins are randomized by thermal fluctuations, and the spontaneous magnetization will vanish at some T_c.

The understand better the space-time picture of chirality, we return to the broken ground state that was made manifest by the 1961 work of Nambu and Jona-Lasinio [2]. Their ground state may be interpreted as the chiral-2 rotation of the usual (chiral invariant) Fock space vacuum [4]

$$|\,vac\rangle = \prod_{p,s} e^{2i\theta_p X_2(\boldsymbol{p})}|0\rangle, \quad (3)$$

where X_2 is one of the generators of the chiral spin algebra. For one flavor, the generators form the simple SU(2) algebra.

Unlike the ferromagnetic case, the $T = 0$ chirality ground state is not given by the spatial ordering of spins, but instead is a distribution of quark-antiquark pairs arising from the chiral-2 rotation. At $T = 0$, the angle of the chiral-2 rotation is neither minimal ($\theta_p = 0$) nor maximal ($\theta_p = \pi/4$), it being related to the mass gap by the relation

$$\tan 2\theta_p = \frac{m}{p}. \quad (4)$$

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As temperature increases, it is up to the dynamics to tell us whether θ_p increases or decreases.

For the NJL model (and its progenitor the BCS model), θ_p indeed decreases with T, so that θ_p vanishes at T_c. In QCD, our calculations show that the corresponding θ_p takes the opposite tack and approaches the maximal $\pi/4$.

3. Dirac Equation at High T

Real time temperature field theory calculations in QCD have shown that the Dirac equation for the fermion propagating in a thermal environ has the generic form

$$\left(\gamma \cdot \nabla A(\nabla, T) - \gamma_0 \frac{\partial}{\partial t} B(\nabla, T)\right)\psi(x) = 0\,, \quad (5)$$

so that the Dirac equation is a spatially non-local one. There is an obvious chiral invariance of this Dirac equation under the global transformation (1). For the ordinary Dirac equation at $T = 0$, such a chiral invariance immediately suggests a massless fermion solution. The solution of our Dirac equation, however, shows a particle propagating through the thermal environ with a Lorentz-invariant mass:

$$\mathcal{M}^2 \overset{T\to\infty}{\longrightarrow} \frac{2\pi^2}{3} \frac{T^2}{\ln \frac{T^2}{\Lambda_c^2}} \left(1 + O\left(\ln \frac{T^2}{\Lambda_c^2}\right)^{-2}\right). \quad (6)$$

In spite of its mass, the solution at the same time also satisfies the property $\langle \bar{\psi}\psi \rangle = 0$.

The vanishing of $\langle \bar{\psi}\psi \rangle$ at high temperature suggests a new chiral invariance associated with (5). The Noether current of the new chiral symmetry, however, is *not* the usual zero temperature Noether current

$$\boldsymbol{Q}_5 = \tfrac{1}{2} \int d^3 x\, \psi^\dagger \gamma_5 \boldsymbol{\tau} \psi \quad (7)$$

but

$$\boldsymbol{Q}_{5T} = \tfrac{1}{4} \int d^3 x\, \psi^\dagger \gamma_5 \boldsymbol{\tau} B(\nabla, T) \psi + \text{h.c.} \quad (8)$$

Above T_c it is the new $\boldsymbol{Q}_{5T}$ that is conserved.

4. Pion Mass

As a follow-up to our earlier work, we have further calculated the pion mass in QCD at high temperature [5].

For this purpose, we let the external sources j and $\boldsymbol{j}$ couple to the fermion bilinears. The Yukawa coupling of these sources to the fermions induce new infinities that are not removed by the counterterms of $\mathcal{L}_{\text{QCD}}$. To remove them in a way that respects renormalization group we have to introduce *ab initio* additional tree-level terms in $\mathcal{L}_J$ so that to each order in the gluon coupling constant $\mathcal{L}_J$ *(counter) cancel* the new infinities. These terms turn the $\mathcal{L}_J$ into a full-blown σ-model tree Lagrangian, after a trivial rescaling of the j and $\boldsymbol{j}_5$ sources:

$$\begin{aligned}\mathcal{L}_\sigma = &-\frac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \boldsymbol{\pi})^2] - h_r(v_o + \sigma\,\mu_\sigma^{\varepsilon/2})\bar{\psi}\psi \\ &- 2i h_r \mu_\sigma^{\varepsilon/2} \boldsymbol{\pi} \cdot \bar{\psi}\, \boldsymbol{T} \gamma_5 \psi - \frac{\kappa_r}{24} \mu_\sigma^{\varepsilon} [(\sigma + \mu_\sigma^{-\varepsilon/2} v_o)^2 + (\boldsymbol{\pi})^2]^2 \\ &+ \frac{v_r^2}{12} [(\sigma + \mu_\sigma^{-\varepsilon/2} v_o)^2 + (\boldsymbol{\pi})^2] + \mathcal{L}_\sigma(counter)\,. \end{aligned} \quad (9)$$

A renormalization group invariant effective potential in σ and $\boldsymbol{\pi}$ may now be defined [7]:

$$\int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}\mathcal{G}_\mu\, e^{i \int d^4 x (\mathcal{L}_{\text{QCD}} + \mathcal{L}_\sigma)} \equiv e^{i\Gamma}\,. \quad (10)$$

Not that the σ and $\boldsymbol{\pi}$ fields are not integrated over on the left hand side. Nevertheless, Γ has all the properties of an effective potential in σ and $\boldsymbol{\pi}$. In the calculation of Γ it is only the quarks and gluons that propagate in the internal loops. In this picture, then, the σ and $\boldsymbol{\pi}$ are not fundamental fields but are genuine bound states of the quark-antiquark system.

Because the $\mathcal{L}_J$ *(counter)* is determined by the renormalizability requirement, the coupling constants in $\mathcal{L}_\sigma$ are not free but are determined in terms of g_r, the gluon coupling constant. This leads to an important difference from the usual linear σ-model. In our dynamically induced σ model (DSM), the coupling constants h_r, κ_r are calculable, being proportional to g_r and g_r^2, respectively, by eigenvalue conditions [8].

At $T = 0$, then, our results is that the effective potential is the dynamically induced σ model, and the pion is easily seen to be massless. This comes about as a result of the cancellation to all orders between the tadpole contribution and the pion self-energy graphs.

5. Restoration at T_c?

At nonzero T, this cancellation continues to occur so long as v, the vacuum shift of the σ field, is not zero. In a perturbative calculation, v to one loop appears to acquire a temperature dependence with the generic form

$$h_r v = h_r v_o \left\{1 - a \frac{T^2}{m_r^2}\right\}, \tag{11}$$

where v_o is the tree level shift as given in $\mathscr{L}_\sigma$.

It is tempting to conclude from this that at T_c, the vacuum shift v vanished. As a result, the delicate cancellation between the tadpole contribution and the pion self-energy graph would be destroyed, and the pion would end up being massive, with mass proportional to T. Generically, that is the origin of the usual conclusion about symmetry restoration at high temperatures.

For our DSM, we can actually perform the higher order calculations using the renormalization group. The higher order graphs involve the so-called daisy and super-daisy graphs [9]. They result her in an alternating series

$$h_r v = h_r v_o \left\{1 - a \frac{T^2}{m_r^2} + b \frac{T^4}{m^4} - \dots\right\} \tag{12}$$

which destabilizes the one loop perturbative root. Indeed, renormalization group summation gives a result that in the end is independent of T, with the high temperature mass of the quark as given by (6).

Dolan-Jackiw [9] already noted the need to look at daisy and super daisy graphs at high temperatures. In their case of a scalar field the daisy graphs to towards stabilizing the one-loop contribution.

The moral here is that symmetry restoration based on one loop arguments need to be checked to higher orders.

6. Lattice Calculations

Our results appear at first sight to be at odds with the lattice calculations. This is decidedly not so. There is no conflict between the analytic field theory calculations and the calculations on the lattice. What is at odds is the conclusion about symmetry restoration that many lattice people draw from their results.

For a major feature of the lattice calculation is the vanishing of $\langle \bar{\psi} \psi \rangle$ for T above T_c. Our calculations show the same feature, with the analytic result that $T_c = \Lambda_c e^{2/3}$, where Λ_c is the $\overline{MS}$ invariant cut-off.

Lattice calculations have also looked for the so-called screening mass of the quarks. This is generically obtained by studying the large z limit of the vacuum expectation value

$$\int \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}t \, \langle \bar{\psi}(0) \gamma_3 \partial_z \psi(x, y, z, t) \rangle \,. \tag{13}$$

Their result shows a screening mass of the quark that is proportional to T.

In the language of the propagator, the screening mass is obtained from the position of the pole when p_x, p_y, p_o have been set equal to zero. At high temperatures, the position of the pole is determined by the equation of the generic form

$$\boldsymbol{p}^2 (1 + A)^2 - p_o^2 (1 + B)^2 = 0\,, \tag{14}$$

where A, B are functions of $\boldsymbol{p}^2, p_o^2, T^2$.

The physical mass is obtained by setting p_o on the mass shell. In general, for a thermal environ, $\mathscr{M}$ is expected to be a function of $\boldsymbol{p}^2$. For QCD (as well as QED), it turn out that $\mathscr{M}$ is Lorentz-invariant [6]. Therefore, the physical mass may be obtained from the root of the equation

$$\mathscr{M}^2 (1 + B(0, \mathscr{M}^2, T^2))^2 = 0 \tag{15}$$

versus the screening mass that is obtained from the root of the equation, where $p_z = -i m_{sc}$,

$$p_z^2 (1 + A(p_z^2, 0, T^2))^2 = 0\,. \tag{16}$$

For the case of the QCD fermion, the two masses turn out to be identical.

7. Pion Halo

For the pion, the corresponding equation reads

$$\boldsymbol{p}^2 (1 + A_\pi)^2 - p_o^2 (1 + B_\pi)^2 = 0\,, \tag{17}$$

and the masslessness follows from the relation

$$A_\pi(\boldsymbol{p}^2, \boldsymbol{p}^2, T^2) = B_\pi(\boldsymbol{p}^2, \boldsymbol{p}^2, T^2)\,. \tag{18}$$

On the other hand, the screening mass that has been found by the lattice calculation derives from the condition

$$p_z^2 (1 + A_\pi(p_z^2, 0, T^2))^2 = 0\,, \tag{19}$$

and there is no conflict between the two conclusions.

In order to better understand how a particle can be massless and still have a non-vanishing screening mass, we will do a sample evaluation of the retarded Lienard-Wichert potential for the pion, with

$$A_\pi = B_\pi = \left(1 + \frac{T^2}{\boldsymbol{p}^2}\right)^{1/2} - 1\,. \tag{20}$$

The retarded Green function for such a pion may be integrated to give ($s \equiv t^2 - r^2$)

$$\Delta_R(r,t) = \frac{\theta(t)}{2\pi}\left\{\delta(s) - \frac{T}{4r}\left[e^{-T|t-r|} - e^{-T|t+r|}\right]\right\}, \quad (21)$$

which is to be compared with the retarded function for a physically massive pion

$$\Delta_R^{(m)} = \frac{\theta(t)}{2\pi}\left\{\delta(s) - \theta(s)\frac{m}{2\sqrt{s}} J_1(m\sqrt{s})\right\}. \quad (22)$$

In the usual Lienard-Wiechert potential, the light signal as seen by the observer at point $\boldsymbol{r}$ came from the single retarded position of the charged particle at earlier time $t = -r$. In contrast, the signal of a truly massive carrier is smeared over all the earlier times $t \leq -r$ of the charged particle.

For the pion in the hot QCD medium, however, the signal propagates along the light cone, with an exponential halo within a skin depth of the light cone. An observer looking at the 'charged' particle through the pion signal would 'see' the charged particle at the retarded position $t = -r$, with an exponentially damped fuzziness associated with signals that come within a skin depth of the earlier position of the charged particle $t = -r \pm 1/T$.

8. Conclusion

A massless pion is the best signal that chiral symmetry remains broken above T_c. And yet there *is* afterall a phase transition at T_c where $\langle\bar{\psi}\psi\rangle$ vanishes. For some speculations, see [10].

It is a pleasure to dedicate this talk to George on his happy 60th.

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